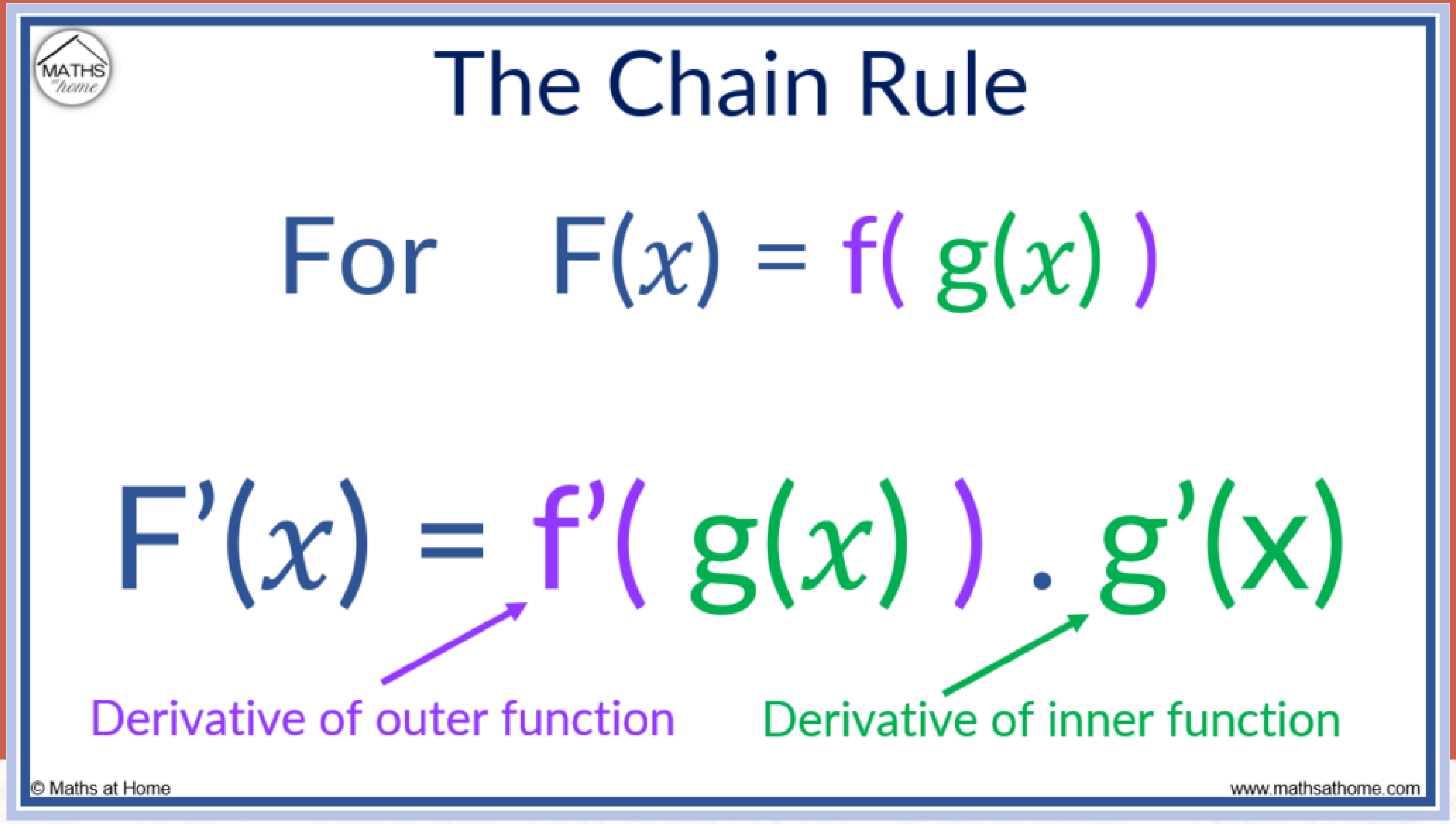
AP Calc Unit 3

www.loopsofkindness.com

Table of Contents

3.1 Chain Rule

- The chain rule makes it easier for us to differentiate composite functions, which are just functions inside other functions!
- To differentiate with the chain rule, you differentiate the outer function while keeping the inner function the same, then multiply this by the derivative of the inner function.



3.1 Chain Rule

A composite function can be written as $w(u(x))$, where u and w are basic functions.

Is $f(x) = \ln(\sin(x))$ a composite function? If so, what are u and w?

A composite function has many little functions within it. Here, ln(sin(x)) is composite since the function sin(x) is inside the function ln(x). $u(x) = \sin(x)$ is the inside function. $w(x) = \ln(x)$, the outside function.

Find $\frac{d}{dx}(\cos(e^x))$.

Chain rule:

$$\frac{d}{dx}\left[v\Big(u(x)\Big)\right] = \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= v'\Big(u(x)\Big) \cdot u'(x)$$

Outside function v is cos(x). Inside function u is e^x.

$$\frac{d}{dx}(\cos(e^x)) = -\sin(e^x) \cdot e^x$$

3.1 Chain Rule

The following table lists the values of functions f and g, and of their derivatives, f' and g', for the x-values -2 and 1.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-2	10	1	-7	4
1	-2	-1	4	5

Let function G be defined as $G(x) = f\left(g(x)\right)$.

$G'(-2) =$ ☐

$$G'(x) = f'(g(x))\,g'(x)$$

$$G'(-2) = f'(g(-2))\,g'(-2) = f'(1)(4) = 4(4) = 16$$

$$\frac{d}{dx}\log_2(2x^2 - 7x + 1).$$

$$\frac{d}{dx}\left[v\left(u(x)\right)\right] = \frac{dv}{du}\cdot\frac{du}{dx}$$

$$= v'\left(u(x)\right)\cdot u'(x)$$

Outside function v is $\log_2 x$, inside function u is $2x^2-7x+1$.

$$\frac{d}{dx}\log_2(2x^2-7x+1) = \frac{1}{(2x^2-7x+1)\ln 2}\cdot 4x-7 = \boxed{\frac{4x-7}{(2x^2-7x+1)\ln 2}}$$

3.2 Implicit Differentiation

- Implicit differentiation is used for functions that can't be put in the form y = f(x) (e.g. $x^2 + y^2 = 9$). These functions are known as implicit.
  - Functions such as y = mx + b are explicit.
- For implicit differentiation, we take the derivative of both sides.
- We differentiate x as we usually do and write dy/dx whenever differentiate y
- Then, solve for dy/dx.

3.2 Implicit Differentiation

$f(x)$

$$x + 2xy - y^2 = 2$$

Find the value of $\frac{dy}{dx}$ at the point $(2, 4)$.

① Take the derivative w/ respect to x.

$$\frac{d}{dx}(x + 2xy - y^2) = \frac{d}{dx}(2)$$

$$\frac{d}{dx}(x) + 2\frac{d}{dx}(xy) - \frac{d}{dx}(y^2) = 0$$

$$1 + 2\left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$$

↑ product rule

↖ chain rule w/ outer function $f(x) = x^2$ and inner function y.

② Substitute (2,4) and solve

$$1 + 2\left(4 + 2\frac{dy}{dx}\right) - 2(4)\frac{dy}{dx} = 0$$

$$1 + 8 + 4\frac{dy}{dx} - 8\frac{dy}{dx} = 0 \Rightarrow -4\frac{dy}{dx} = -9 \Rightarrow \boxed{\frac{dy}{dx} = \frac{9}{4}}$$

3.3 Differentiating Inverse Functions

- An inverse function essentially just flips the coordinates. If g is the inverse of f, and f exists at (3, 2), then g will exist at (2,3)
- There are two formulas for this, but they both mean the same thing. Let g be the inverse of f:

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$$

$$g'(x) = \frac{1}{f'(g(x))}$$

3.3 Differentiating Inverse Functions

Let $f(x) = x^3 + 2x - 1$ and let g be the inverse function of f. Notice that $f(2) = 11$.

$g'(11) = $ ☐

$$g'(x) = \frac{1}{f'\left(g(x)\right)}$$

$$g'(11) = \frac{1}{f'(g(11))}$$

(swapped x and y)

If g is the inverse of f, $g(11) = 2$. We substitute $g(11) = 2$

$$g'(11) = \frac{1}{f'(g(11))} = \frac{1}{f'(2)} = \boxed{\frac{1}{14}}$$

$f'(x) = 3x^2 + 2$ $\quad f'(2) = 3(4) + 2 = 12 + 2 = 14$

3.4 Differentiating Inverse Trig Funcs

- This topic is mostly just memorization of these derivatives.

Inverse Trigonometric Derivatives

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\text{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

3.5 Selecting Procedures for Derivatives

Here are the procedures we've learned to find the derivative!

Unit 2

- Power rule
- Derivative of constants
- Constant multiple
- Sum/Difference
- Derivatives of trig functions
- Derivatives of exponents
- Derivatives of a logarithm
- Product rule
- Quotient rule

Unit 3

- Chain rule
- Implicit differentiation
- Inverse functions
- Inverse trig

3.6 Calculating Higher-Order Derivatives

- To find the second derivative, we just take the derivative twice. To find the third derivative, we take the derivative 3 times and so on.
-

Second Derivative: $\frac{d^2y}{dx^2}, \; f''(x), \; f^{(2)}(x)$

Third Derivative: $\frac{d^2y}{dx^3}, \; f'''(x), \; f^{(3)}(x)$

n^{th}-Order Derivative: $\frac{d^ny}{dx^n}, \; f^{(n)}(x)$

3.6 Calculating Higher-Order Derivatives

Find $\frac{d^2}{dx^2}[6\cos(3x+3)]$.

First derivative is $\frac{d}{dx}[6\cos(3x+3)] = -6\sin(3x+3)\cdot 3$

$= -18\sin(3x+3)$

Second derivative is $\frac{d}{dx}[-18\sin(3x+3)] = -18\cos(3x+3)\cdot 3 = \boxed{-54\cos(3x+3)}$

Learn to crochet and get community service hours now!

Learn more at www.loopsofkindness.com/get-involved